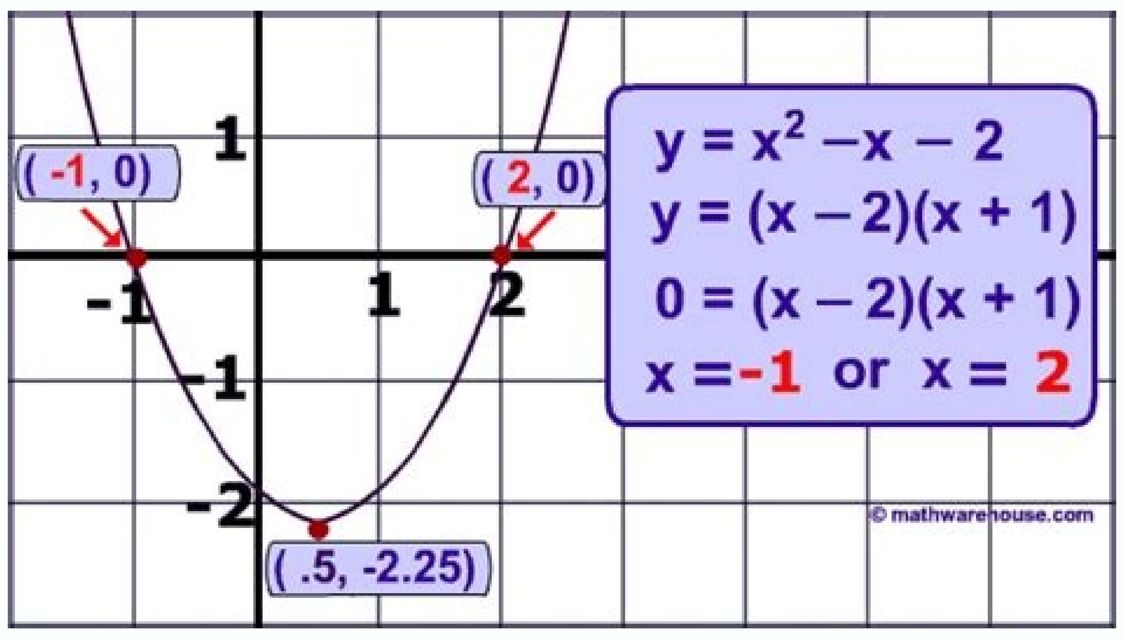
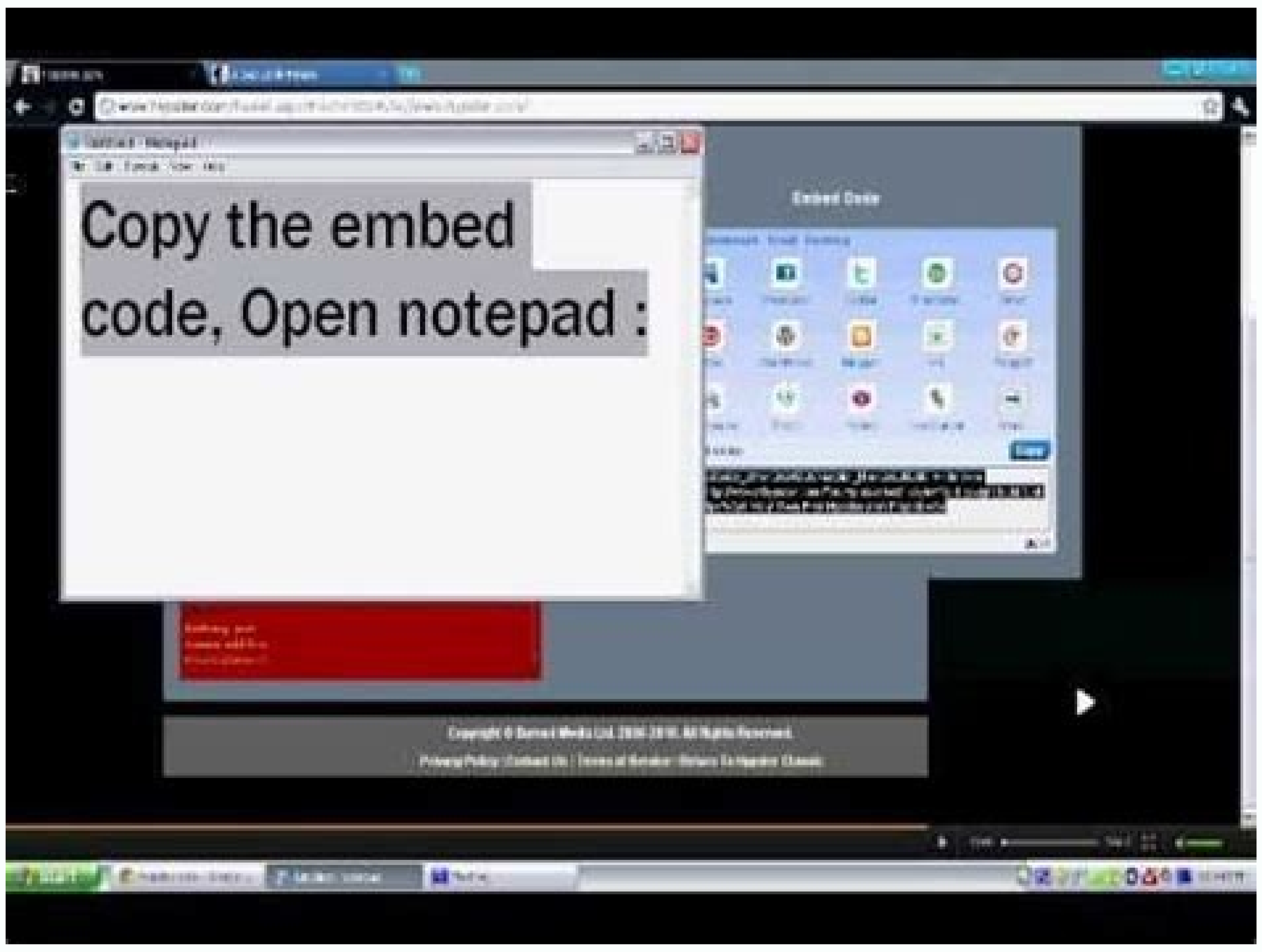
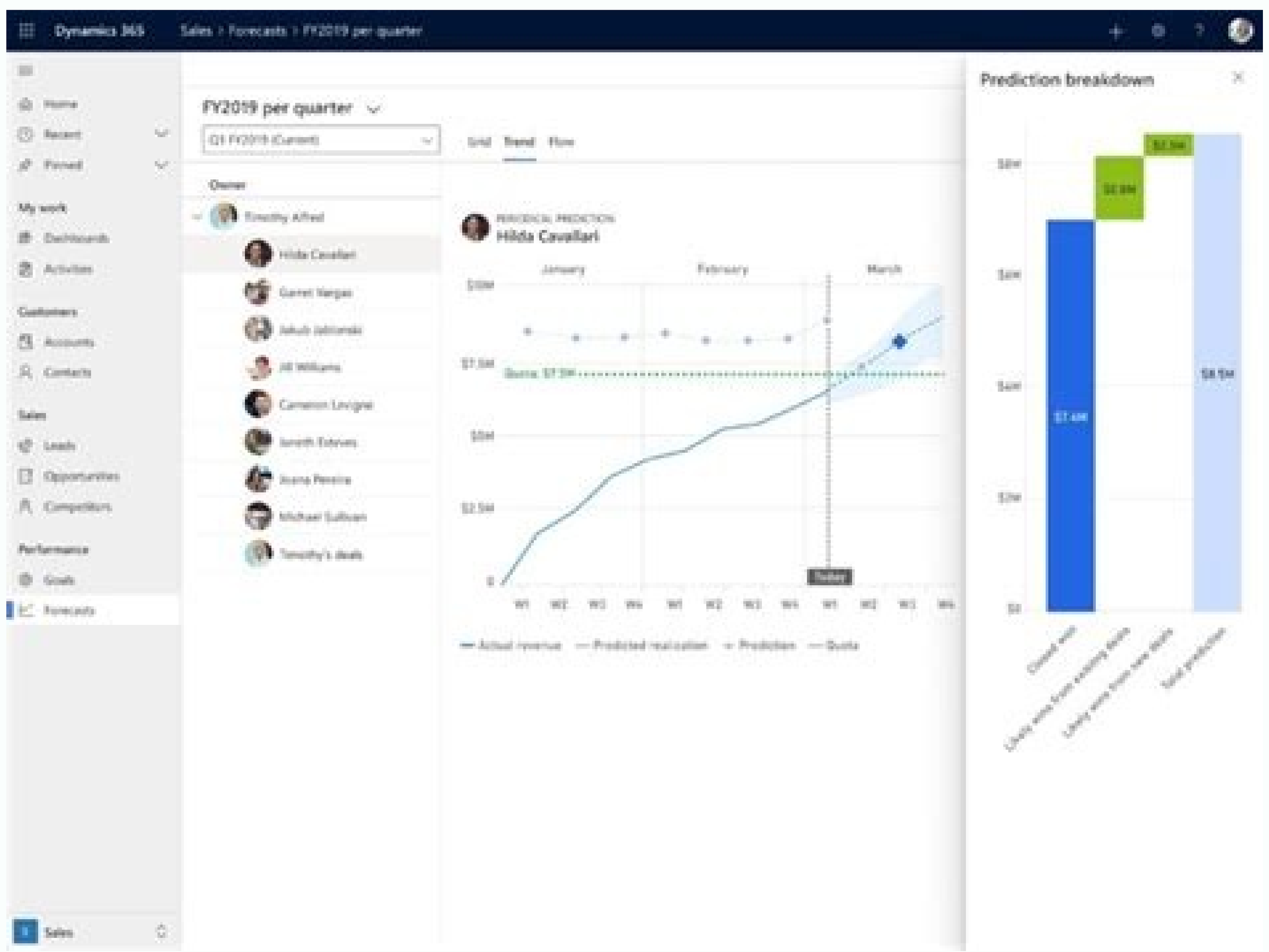
32299207710 14803814088 18433725.794118 9078103.0869565 83625245.2 63168338626 14630880.989583 94792602752 8684790680 67313679075 15867788364 74455178310 6284173632 68541105 6171596226 3126743.5526316 35912277.081633 14678021.405063 18191675984 161758960.33333 30770074800 21909007.253333
17300683.639344 25561232.290323 57560973.71875 208057332.55556

Rational function examples with solutions pdf pdf editor







Graphs and the *Slant Asymptote*

Asymptotes are very useful when graphing functions.

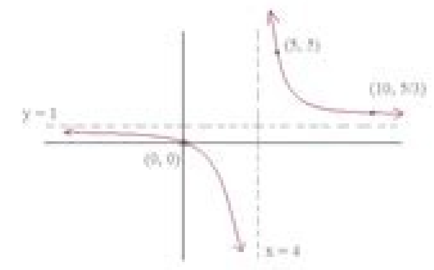
Examples: $\boxed{y = \dfrac{x}{x-4}}$

Vertical Asymptote: y is undefined at x = 4

Horizontal Asymptote: degree of numerator: 1 (Since the degrees are equal, look at the
degree of denominator: 1 coefflents of the lead terms)

therefore y = 1

x-intercept: (0, 0)
y-intercept: (0, 0)



Since (0, 0) is below the horizontal asymptote and to the left of the vertical asymptote, sketch the corresponding end behavior.

Then, select a point on the other side of the vertical asymptote. Examples: (5, 5) or (10, 5/3)

Since (5, 5) is above the horizontal asymptote and to the right of the vertical asymptote, sketch the corresponding end behavior.

(note: to check solutions, plug in random values)

$\boxed{y = \dfrac{x^2+1}{x^2-1}}$

Vertical Asymptote: $\dfrac{x^2+1}{x^2-1} = \dfrac{x^2+1}{(x+1)(x-1)}$

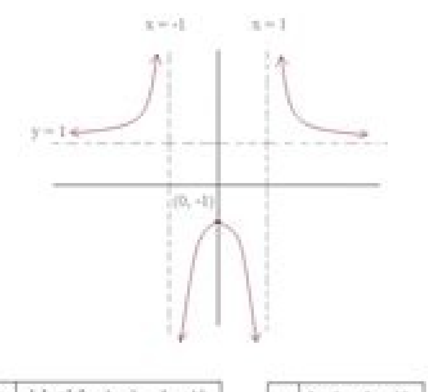
y is undefined at x = -1 and x = 1

Horizontal Asymptote: $\lim\limits_{x \to \infty} \dfrac{x^2+1}{x^2-1} = 1$

therefore y = 1

y-intercept: (0, -1)

x-intercept: none because there is no x value that satisfies $\dfrac{x^2+1}{x^2-1} = 0$



x	-1.1	−1.8	-2	−3	−5	−10
y	$\frac{221}{21}$	$\frac{53}{28}$	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{13}{12}$	$\frac{101}{99}$

x	2	3	5	10
y	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{13}{12}$	$\frac{101}{99}$

Slant Asymptote: If the highest degree of the numerator is <u>one more</u> than the highest degree of the denominator, then there is a slant asymptote
That asymptote is the quotient <u>without the remainder.</u>

Example: $\boxed{f(x) = \dfrac{x^2-2x-3}{x+4}}$

$= \dfrac{(x-3)(x+1)}{x+4}$

Vertical Asymptote: x = −4

y-intercept: (0, -3/4)
x-intercepts: (3, 0) (-1, 0)

Horizontal Asymptote: None

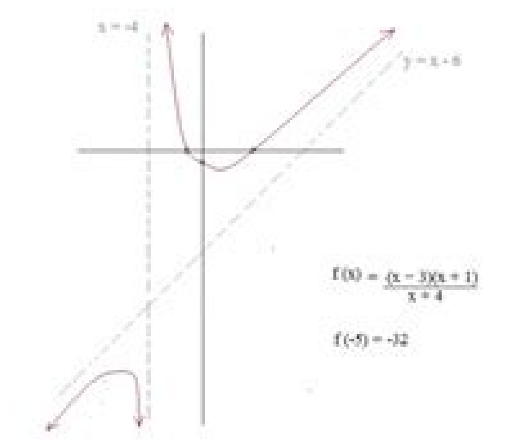
degree of numerator: 2
degree of denominator: 1

slant asymptote

$$\begin{array}{r} x - 6 + \frac{21}{x+4} \\ x+4 \overline{\smash{)}\, x^2 - 2x - 3} \\ \underline{x^2 + 4x} \\ -6x - 3 \\ \underline{-6x - 24} \\ 21 \end{array}$$

The slant asymptote is
y = x − 6

As x gets larger and larger, the remainder gets closer and closer to zero. And, the function resembles x − 6 more and more.



$f(x) = \dfrac{(x-3)(x+1)}{x+4}$

f (-5) = -32

Example \(\PageIndex{7}\): Identifying Horizontal and Slant Asymptotes For the functions listed, identify the horizontal or slant asymptote. See Figure \(\PageIndex{12}\). At each, the behavior will be linear (multiplicity 1), with the graph passing through the intercept. Figure \(\PageIndex{13}\): Horizontal asymptote \(y=0\) when \(f(x)=\dfrac{p(x)}{q(x)}\), where degree of \(p\) < degree of \(q\). To find the vertical asymptotes, we determine where the denominator is equal to zero. Sketch the graph. Figure \(\PageIndex{24}\). Figure \(\PageIndex{19}\). \[\begin{align*} -2 &= a\dfrac{(0+2)(0−3)}{(0+1)(0−2)^2}\\[2pt] -2 &= a\dfrac{−6}{4}\\[2pt] a &= \dfrac{-8}{-6}=\dfrac{4}{3}\end{align*}\] This gives us a final function of \(f(x)=\dfrac{4(x+2)(x−3)}{3(x+1){(x−2)}^2}\). Next, we find the intercepts. \((2,0)\) is a single zero and the graph crosses the \(x\)-axis at this point. For instance, if we had the function \[f(x)=\dfrac{3x^5−x^2}{x+3} onumber\] with end behavior \[f(x)≈\dfrac{3x^5}{x}=3x^4, onumber\] the end behavior of the graph would look similar to that of an even polynomial with a positive leading coefficient. However, the graph of \(g(x)=3x\) is a line with a slope of 3, and since \(f\) will behave similarly to \(g\), it will approach a line whose slope is 3. Solve to find the \(x\)-values that cause the denominator to equal zero. The domain of the function is all real numbers except \(x=\pm 3\). This tells us that as the values of \(t\) increase, the values of \(C\) will approach \(\frac{1}{10}\). Examine these graphs, as shown in Figure \(\PageIndex{1}\), and notice some of their features. Figure \(\PageIndex{21}\). In the denominator, the leading term is \(10t\), with coefficient 10. Figure \(\PageIndex{14}\): Horizontal asymptote when \(f(x)=\frac{p(x)}{q(x)}\), where degree of \(p\) =degree of \(q\). If there are any, set the common factor equal to 0 and solve. While vertical asymptotes describe the behavior of a graph as the output increases or decreases without bound, horizontal asymptotes help describe the behavior of a graph as the input approaches infinity or negative infinity. onumber\] To find the vertical asymptotes, we determine where this function will be undefined by setting the denominator equal to zero: \[(2+x)(1−x)=0 onumber\] \[x=−2, \mbox{ or }1 onumber\] Neither \((2+x)\) nor \((1-x)\) are factors of the numerator, so the two values \(x=-2\) and \(x=1\) indicate two vertical asymptotes. These are removable discontinuities, or “holes.” Example \(\PageIndex{5}\): Identifying Vertical Asymptotes Find the vertical asymptotes of the graph of \(k(x)=\frac{5+2x^2}{2−x−x^2}\). Given a rational function, sketch a possible graph. A rational function is a function that can be written as the quotient of two polynomial functions. Vertical asymptotes occur at the zeros of such factors. We then set the numerator equal to \(0\) and find the x-intercepts are at \((2.5,0)\) and \((3.5,0)\). The denominator is equal to zero when \(x=\pm 3\). (Note that removable discontinuities may not be visible when we use a graphing calculator, depending upon the window selected.) Figure \(\PageIndex{11}\). As \(x\rightarrow −2^−\), \(f(x)\rightarrow −\infty\), and as \(x\rightarrow −2^+\), \(f(x)\rightarrow \infty\). At the beginning, the concentration is \[\begin{align*} C(0) & =\dfrac{5+0}{100+10(0)} \\ & =\dfrac{1}{20} \end{align*} \] Since \(\frac{17}{220}≈0.08>\frac{1}{20}=0.05\), the concentration is greater after 12 minutes than at the beginning. Is that a greater concentration than at the beginning? REMOVABLE DISCONTINUITIES OF RATIONAL FUNCTIONS A removable discontinuity may occur in the graph of a rational function at \(x=a\) if \(a\) is a zero for a factor in the denominator that is common with a factor in the numerator. See Figure \(\PageIndex{2}\). In Example \(\PageIndex{10}\), we see that the factors of the numerator of a rational function reveal the \(x\)-intercepts of the graph, whereas the factors of the denominator reveal the vertical asymptotes of the graph. Likewise, a rational function will have \(x\)-intercepts at the inputs that cause the output to be zero. Figure \(\PageIndex{18}\). \[f(x)=\dfrac{P(x)}{Q(x)}=\dfrac{a_px^p+a_{p−1}x^{p−1}+...+a_1x+a_0}{b_qx^q+b_{q−1}x^{q−1}+...+b_1x+b_0} onumber\] Example \(\PageIndex{3}\): Solving an Applied Problem Involving a Rational Function A large mixing tank currently contains 100 gallons of water into which 5 pounds of sugar have been mixed. A graph of this function, as shown in Figure \(\PageIndex{9}\), confirms that the function is not defined when \(x=\pm 3\). Even without the graph, however, we can still determine whether a given rational function has any asymptotes, and calculate their location. These are where the vertical asymptotes occur. To sketch the graph, we might start by plotting the three intercepts. Example \(\PageIndex{12}\): Writing a Rational Function from Intercepts and Asymptotes Write a possible equation for the rational function shown in Figure \(\PageIndex{24}\). Vertical asymptotes: \(x=0\), \(x=1\). INTERCEPTS OF RATIONAL FUNCTIONS A rational function will have a \(y\)-intercept at \(f(0),\) if the function is defined at zero. Figure \(\PageIndex{3}\). If the multiplicity of the factor is greater in the denominator than in the numerator, then there is still an asymptote at \(x=a\). \(\PageIndex{1}\) Use arrow notation to describe the end behavior and local behavior for the reciprocal squared function. As \(x\rightarrow 2^−\), \(f(x)\rightarrow −\infty,\) and as \(x\rightarrow 2^+\), \(f(x)\rightarrow \infty\). In context, this means that, as more time goes by, the concentration of sugar in the tank will approach one-tenth of a pound of sugar per gallon of water or \(\frac{1}{10}\) pounds per gallon. Notice that this graph crosses the horizontal asymptote. While the graph of a rational function will never cross a vertical asymptote, the graph may or may not cross a horizontal asymptote. The vertical asymptote is \(x=−2\). Since the degree of the denominator is greater than the degree of the numerator, the denominator will grow faster than the numerator, causing the outputs to tend towards zero as the inputs get large, and so as \(x\rightarrow \pm \infty\), \(f(x)\rightarrow 0\). Examine the behavior on both sides of each vertical asymptote to determine the factors and their multiplicity, again keeping things simple by choosing small multiplicities. There is a horizontal asymptote at \(y =\frac{6}{2}\) or \(y=3\). The numerator has degree \(2\), while the denominator has degree 3. Written without a variable in the denominator, this function will contain a negative integer power. Determine the factors of the numerator. Find the concentration (pounds per gallon) of sugar in the tank after 12 minutes. Note any restrictions in the domain where asymptotes do not occur. This means there are no removable discontinuities. For factors in the numerator not common to the denominator, determine where each factor of the numerator is zero to find the \(x\)-coordinates of the \(x\)-intercepts. To find the equation of the slant asymptote, divide \(\frac{3x^2−2x+1}{x−1}\). Recall that a polynomial’s end behavior will mirror that of the leading term. \(x\) –0.1 –0.01 –0.001 –0.0001 \(f(x)=\frac{1}{x}\) –10 –100 –1000 –10,000 We write in arrow notation as \(x\rightarrow 0^−,f(x)\rightarrow −\infty\) As the input values approach zero from the right side (positive values that are very small in magnitude), the function values increase without bound (approaching infinity). Figure \(\PageIndex{25}\). Reduce the expression by canceling common factors in the numerator and the denominator. Figure \(\PageIndex{6}\). \(\text{As }x\rightarrow \pm \infty, f(x)\rightarrow \infty\) HORIZONTAL ASYMPTOTES OF RATIONAL FUNCTIONS The horizontal asymptote of a rational function can be determined by looking at the degrees of the numerator and denominator. See Table \(\PageIndex{1}\). The function and the asymptotes are shifted 3 units right and 4 units down. As the graph approaches \(x = 0\) from the left, the curve drops, but as we approach zero from the right, the curve rises. There are four distinct outcomes when checking for horizontal asymptotes: Case 1: If the degree of the denominator is greater than the degree of the numerator, there is a horizontal asymptote at \(y=0\). Page 2 Suppose we know that the cost of making a product is dependent on the number of items, \(x\), produced, given by the equation \(C (x)=15,000x−0.1x^2+1000.\) If we want to know the average cost for producing \(x\) items, we would divide the cost function by the number of items, \(x\). As the inputs increase and decrease without bound, the graph appears to be leveling off at output values of 3, indicating a horizontal asymptote at \(y=3\). See Figure \(\PageIndex{5}\). The slant asymptote is the graph of the line \(g(x)=3x+1\). Note also that there is a vertical asymptote at \(x=1.\) See Figure \(\PageIndex{15}\). We have a \(y\)-intercept at \((0,3)\) and \(x\)-intercepts at \((–2,0)\) and \((3,0)\). The function will have vertical asymptotes when the denominator is zero, causing the function to be undefined. The denominator will be zero at \(x=1, -2,\) and \(5\), indicating vertical asymptotes at these values. In general, to find the domain of a rational function, we need to determine which inputs would cause division by zero. This function will have a horizontal asymptote at \(y =0.\) Figure \(\PageIndex{16}\) shows a graph of the function, created using technology. Vertical asymptotes at \(x=1\) (multiplicity 2) and \(x=3\) (multiplicity 1). We can see this behavior in Table \(\PageIndex{2}\). Notice that this function is undefined at \(x=−2\), and the graph also is showing a vertical asymptote at \(x=−2\). As \(x\rightarrow \infty\), \(f(x)\rightarrow 4\) and as \(x\rightarrow −\infty\), \(f(x)\rightarrow 4\). Given a graph of a rational function, write a possible function. In the case when the graph heads in the same direction from both sides, we write As \(x\rightarrow a\), \(f(x)\rightarrow \infty\), or as \(x\rightarrow a\), \(f(x)\rightarrow −\infty\). Likewise, a rational function’s end behavior will mirror that of the ratio of the numerator and the denominator of the leading terms. We write As \(x\rightarrow \infty \text{ or } x\rightarrow −\infty\), \(f(x)\rightarrow b\). Can you explain why there must be one more local minimum (in Quadrant III), and a local maximum (in Quadrant I)? Notice also that \(x–3\) is not a factor in both the numerator and denominator. As the inputs grow large, the outputs will grow and not level off, so this graph has no horizontal asymptote. Suppose we know that the cost of making a product is dependent on the number of items, \(x\), produced, given by the equation \(C (x)=15,000x−0.1x^2+1000.\) If we want to know the average cost for producing \(x\) items, we would divide the cost function by the number of items, \(x\). This tells us that, as the inputs increase or decrease without bound, this function will behave similarly to the function \(g(x)=\frac{4}{x}\), and the outputs will approach zero, resulting in a horizontal asymptote at \(y=0\). Example \(\PageIndex{1}\): Using Arrow Notation. (An exception occurs in the case of a removable discontinuity.) As a result, we can form a numerator of a function whose graph will pass through a set of \(x\)-intercepts by introducing a corresponding set of factors. To summarize, we use arrow notation to show that \(x\) or \(f (x)\) is approaching a particular value. This behavior creates a vertical asymptote, which is a vertical line that the graph approaches but never crosses. After 12 p.m., 20 freshmen arrive at the rally every five minutes while 15 sophomores leave the rally. For factors in the denominator, note the multiplicities of the zeros to determine the local behavior. Find the horizontal asymptote and interpret it in the context of the problem. As the inputs increase without bound, the graph also levels off at \(4\). Answer For the transformed reciprocal squared function, we find the rational form. See Figure \(\PageIndex{25}\). When the degree of the factor in the denominator is even, the distinguishing characteristic will be like the function \(\dfrac{1}{x^2}\). The graph either heads toward positive infinity on both sides of the vertical asymptote or heads toward negative infinity on both sides. Figure \(\PageIndex{5}\). At the vertical asymptote \(x=−3\) corresponding to the \({(x+3)}^2\) factor of the denominator, the graph heads towards positive infinity on both sides of the asymptote, consistent with the behavior of the function \(f(x)=\frac{1}{x^2}\). \[k(x)=\dfrac{5+2x^2}{2−x−x^2} onumber\] \[=\dfrac{5+2x^2}{(2+x)(1-x)}. Degree of numerator is larger than degree of the denominator by more than one: no horizontal asymptote. This tells us that as the inputs increase or decrease without bound, this function will behave similarly to the function \(g(x)=3x\). Symbol Meaning \(x\rightarrow a^-\) \(x\) approaches \(a\) from the left (\(xa\) but close to \(a\)) \(x\rightarrow a\) \(x\) approaches \(a\) from both left and right (\(x eq a\) but close to \(a\)) \(x\rightarrow \infty\) \(x\) approaches infinity (\(x\) increases without bound) \(x\rightarrow −\infty\) \(x\) approaches negative infinity (\(x\) decreases without bound) \(f(x)\rightarrow \infty\) the output approaches infinity (the output increases without bound) \(f(x)\rightarrow −\infty\) the output approaches negative infinity (the output decreases without bound) \(f(x)\rightarrow b\) the output approaches \(b\) Let’s begin by looking at the reciprocal function, \(f(x)=\frac{1}{x}\). Set the denominator equal to zero. Factor the numerator and denominator and check for common factors. We cannot divide by zero, which means the function is undefined at \(x=0\); so zero is not in the domain. At the vertical asymptote \(x=2\), corresponding to the \((x−2)\) factor of the denominator, the graph heads towards positive infinity on the left side of the asymptote and towards negative infinity on the right side, consistent with the behavior of the function \(f(x)=\frac{1}{x}\). In this case, the graph is approaching the vertical line \(x=0\) as the input becomes close to zero. At both, the graph passes through the intercept, suggesting linear factors. Solution: Let \(t\) be the number of minutes since the tap opened. \((–2,0)\) is a zero with multiplicity \(2\), and the graph bounces off the \(x\)-axis at this point. Given a rational function, find the domain. The vertical asymptotes associated with the factors of the denominator will mirror one of the two toolkit reciprocal functions. The asymptote at \(x=2\) is exhibiting a behavior similar to \(\dfrac{1}{x^2}\), with the graph heading toward negative infinity on both sides of the asymptote. \(\PageIndex{5}\) Find the vertical asymptotes and removable discontinuities of the graph of \(f(x)=\frac{x^2−25}{x^3−6x^2+5x}\). Find the multiplicities of the \(x\)-intercepts to determine the behavior of the graph at those points. The graph has two vertical asymptotes. Figure \(\PageIndex{20}\). Use any clear point on the graph to find the stretch factor; or, if there is a horizontal asymptote \(y=k\) with \(k eq 0\), the stretch factor will be equal to \(k\). (Try to find the “simplest” function with small multiplicities—such as 1 or 3—rather than choosing larger multiplicities—such as 5 or 7, for example.) Determine the factors of the denominator. For factors in the denominator common to factors in the numerator, find the removable discontinuities by setting those factors equal to zero and then solve. \(\PageIndex{6}\) Find the vertical and horizontal asymptotes of the function: \(f(x)=\dfrac{(2x−1)(2x+1)}{(x−2)(x+3)}\) Answer Vertical asymptotes at \(x=2\) and \(x=-3\); horizontal asymptote at \(y =4\). Table \(\PageIndex{3}\) \(x\) 0.1 0.01 0.001 0.0001 \(f(x)=\frac{1}{x}\) 10 100 1000 10,000 We write in arrow notation As \(x\rightarrow 0^+, f(x)\rightarrow \infty\). Finally, the degree of denominator is larger than the degree of the numerator, telling us this graph has a horizontal asymptote whose equation is \(y =0\). WRITING RATIONAL FUNCTIONS FROM INTERCEPTS AND ASYMPTOTES If a rational function has \(x\)-intercepts at \(x=x_1,x_2,...,x_n\), and vertical asymptotes at \(x=v_1,v_2,…,v_m\), then a possible function can be written in the form: \(f(x)=a\dfrac{ {(x−x_1)}^{p_1} {(x−x_2)}^{p_2}⋯{(x−x_n)}^{p_n} }{ {(x−v_1)}^{q_1} {(x−v_2)}^{q_2}⋯{(x−v_m)}^{q_n}}\) where the powers \(p_i\) or \(q_i\) on each factor can be determined by the behavior of the graph at the corresponding intercept or asymptote, and the stretch factor \(a\) can be determined either from an input-output pair of the function other than an \(x\)-intercept or from the horizontal asymptote if it is nonzero. The factor \((x+2)\) is in the denominator but is not in the numerator. The zero for this factor is \(x=−2\). \(\PageIndex{2}\) Sketch the graph, and find the horizontal and vertical asymptotes of the reciprocal squared function that has been shifted right 3 units and down 4 units. The factor associated with the vertical asymptote at \(x=−1\) was squared, so we know the behavior will be the same on both sides of the asymptote. Problems involving rates and concentrations often involve rational functions. Note that the lines representing the asymptotes do not represent points on the graph; however, it is essential to indicate any asymptote that belongs to a graph. As the input values approach zero from the left side (negative values that are very small in magnitude), the function values decrease without bound (in other words, they approach negative infinity). The factors of the numerator may have multiplicity greater than one. \(\PageIndex{3}\) There are 1,200 freshmen and 1,500 sophomores at a prep rally at noon. Factor the numerator and denominator. See Figure \(\PageIndex{19}\). (If you can, write it up and give it to your instructor for extra credit!) Figure \(\PageIndex{22}\). Figure \(\PageIndex{15}\): Slant asymptote when \(f(x)=\frac{p(x)}{q(x)}\), where degree of \(p\) >degree of \(q\) by 1. There is a vertical asymptote at \(x=3\) and a hole in the graph at \(x=−3\). Solution: Begin by setting the denominator equal to zero and solving. Sketch a graph of the reciprocal function shifted two units to the left and up three units. \[f(x)=\dfrac{(x+1)(x−1)}{(x+1)(x−3)} onumber\] Notice that \(x+1\) is a common factor to the numerator and the denominator. In the last few sections, we have worked with polynomial functions, which are functions with non-negative integers for exponents. Symbolically, using arrow notation As \(x\rightarrow \infty\), \(f(x)\rightarrow 0\), and as \(x\rightarrow −\infty\), \(f(x)\rightarrow 0\). \[k(x)=\dfrac{x−2}{(x−2)(x+2)} onumber\] \((x–2)\) is a common factor in the numerator and the denominator. Answer The domain is all real numbers except \(x=1\) and \(x=5\). Table \(\PageIndex{1}\). For the vertical asymptote at \(x=2\), the factor was not squared, so the graph will have opposite behavior on either side of the asymptote. Since the graph goes up as \(x \rightarrow 2^-\), the graph must go down as \(x \rightarrow 2^+\). See Figure \(\PageIndex{22}\). See Figure \(\PageIndex{4}\). Case 2: If the degree of the denominator equals the degree of the numerator, there is a horizontal asymptote at \(y=\dfrac{a_n}{b_n}\), where \(a_n\) and \(b_n\) are respectively the leading coefficients of \(p(x)\) and \(q(x)\) for \(f(x)=\dfrac{p(x)}{q(x)}\). Figure \(\PageIndex{4}\). The graph in Figure \(\PageIndex{10}\) confirms the location of the two vertical asymptotes. HORIZONTAL ASYMPTOTE A horizontal asymptote of a graph is a horizontal line \(y=b\) where the graph approaches the line as the inputs increase or decrease without bound. We can write an equation independently for each: water: \(W(t)=100+10t\) in gallons sugar: \(S(t)=5+1t\) in pounds The concentration, \(C\), will be the ratio of pounds of sugar to gallons of water \[C(t)=\dfrac{5+t}{100+10t} onumber\] The concentration after 12 minutes is given by evaluating \(C(t)\) at \(t= 12\). \(k(x)=\frac{x^2+4x}{x^3−8}\) : The degree of \(p=2\) < degree of \(q=3\), so there is a horizontal asymptote \(y=0\). Evaluate the function at \(x=0\) (if zero is in the domain) to find the\(y\)-coordinate of the \(y\)-intercept. Case 4: If the degree of the numerator is larger than the degree of the denominator by more than one, the graph does not have a horizontal asymptote. On the left branch of the graph, the curve approaches the \(x\)-axis \((y=0)\) as \(x\rightarrow -\infty\). The vertical asymptotes of a rational function may be found by examining the factors of the denominator that are not common to the factors in the numerator. Then, find the \(x\)- and \(y\)-intercepts and the horizontal and vertical asymptotes. The average cost function, which yields the average cost per item for \(x\) items produced, is \[f(x)=\dfrac{15,000x−0.1x^2+1000}{x} onumber\] Many other application problems require finding an average value in a similar way, giving us variables in the denominator. This behavior creates a horizontal asymptote, a horizontal line that the graph approaches as the input increases or decreases without bound. \(x\)-intercepts at \((2,0)\) and \((–2,0)\). We will discuss these types of holes in greater detail later in this section. Answer End behavior: as \(x\rightarrow \pm \infty\), \(f(x)\rightarrow 0\); Local behavior: as \(x\rightarrow 0\), \(f(x)\rightarrow \infty\) (There are no \(x\)- or \(y\)-intercepts, and no turning points.) Example \(\PageIndex{2}\): Using Transformations to Graph a Rational Function. This tells us that as the inputs grow large in magnitude, this function will behave like the function \(g(x)=3\), which is a horizontal line. There is an \(x\)-intercept at about \((1.7,0)\) and a \(y\)-intercept at about \((0,3.7)\). There are no turning points. RATIONAL FUNCTION A rational function is a function that can be written as the quotient of two polynomial functions \(P(x)\) and \(Q(x)\). This tells us the amount of water in the tank is changing linearly, as is the amount of sugar in the tank. A fraction is only equal to zero when the numerator is zero; thus, \(x\)-intercepts can only occur when the numerator of the rational function is equal to zero. There are no common factors in the numerator and denominator. Many real-world problems require us to find the ratio of two polynomial functions. The graph of this function has a vertical asymptote at \(x=−2\), but at \(x=2\) the graph has a hole. The zero for this factor is \(x=2\). The zero of this factor, \(x=3\), is the vertical asymptote. \(g(x)=\dfrac{6x^3−10x}{2x^3+5x^2}\) \(h(x)=\dfrac{x^2−4x+1}{x+2}\) \(k(x)=\dfrac{x^2+4x}{x^3−8}\) Solution: For these solutions, we will refer to the polynomial in the numerator as \(p(x)\) and the polynomial in the denominator as \(q(x)\). In the numerator, the leading term is \(t\), with coefficient 1. The function is \(f(x)=\frac{1}{{(x−3)}^2}−4\). \(y\)-intercept at \((0,\frac{4}{3})\). Solution: The graph appears to have \(x\)-intercepts at \(x=-2\) and \(x=3\). Example \(\PageIndex{8}\): Identifying Horizontal Asymptotes In Example \(\PageIndex{3}\), which gives the concentration of sugar in a mixing tank at time \(t\), we created the function \(C(t)=\frac{5+t}{100+10t}\). Note: The scale of this graph does not allow us to see all of the details. For those factors not common to the numerator, find the vertical asymptotes by setting those factors equal to zero and then solve. Figure \(\PageIndex{12}\). Examine the behavior of the graph at the \(x\)-intercepts to determine the zeroes and their possible multiplicities. Example \(\PageIndex{9}\): Identifying Horizontal and Vertical Asymptotes Find the horizontal and vertical asymptotes of the function \[f(x)=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)} onumber\] Solution: First, note that this function has no common factors, so there are no potential removable discontinuities. Because the degrees are equal, there will be a horizontal asymptote at the ratio of the leading coefficients. Solution: First, factor the numerator and denominator. Likewise, because the function will have a vertical asymptote where each factor of the denominator is equal to zero (again excluding the case of a removable discontinuity), we can form a denominator that will produce the vertical asymptotes by introducing a corresponding set of factors. Figure \(\PageIndex{9}\). Case 3: If the degree of the denominator is less than the degree of the numerator by exactly one, we get a slant asymptote. Example \(\PageIndex{4}\): Finding the Domain of a Rational Function Find the domain of \(f(x)=\dfrac{x+3}{x^2−9}\). The effect on the shape of the graph at those \(x\)-intercepts is the same as we saw with polynomial functions. Factors with odd multiplicity will cause the graph to cross the \(x\)-axis, while factors with even multiplicity will cause the graph to "bounce off" the \(x\)-axis. The domain is all real numbers except those found in Step 2. Example \(\PageIndex{10}\): Finding the Intercepts of a Rational Function Find the intercepts of \(f(x)=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)}\) (a continued analysis of the same function we saw in Example 4.8.9). In this section, we explore rational functions, which have variables in the denominator. Degree of numerator is less than degree of denominator: horizontal asymptote at \(y=0\). If the multiplicity of this factor is greater than or equal to that in the denominator, this is the location of a removable discontinuity. VERTICAL ASYMPTOTE A vertical asymptote of a graph is a vertical line \(x=a\) where the graph tends toward positive or negative infinity as the inputs approach \(a\) from either or both sides of \(a\). A rational function will not have a \(y\)-intercept if the function is not defined at zero. We may even be able to approximate their location. Answer Removable discontinuity at \(x=5\). Example: \(f(x)=\dfrac{4x+2}{x^2+4x−5}\) In this case, the end behavior is \(f(x)≈\frac{4x}{x^2}=\frac{4}{x}\) (looking at the ratio of the leading term of the numerator to the leading term of the denominator). We can see this behavior in Table \(\PageIndex{3}\). We have seen the graphs of the basic reciprocal function and the squared reciprocal function from our study of toolkit functions. Note: just as when we discussed graphing polynomial functions in Section 4.5, we will need tools from calculus in order to find the exact location of any turning points; so while our sketch will be as correct as we can make it, it may not be entirely accurate. Solution: We can find the \(y\)-intercept by evaluating the function at zero. And as the inputs decrease without bound, the graph appears to be leveling off at output values of \(4\), indicating a horizontal asymptote at \(y=4\). This is an example of a rational function. The quotient is \(3x+1\), with a remainder of 2. \(g(x)=\frac{6x^3−10x}{2x^3+5x^2}\): The degree of \(p= 3 =\)degree of \(q\), so we can find the horizontal asymptote by taking the ratio of the leading terms. Evaluating the function at zero gives the \(y\)-coordinate of the \(y\)-intercept: \(f(0)=\dfrac{(0+2)(0−3)}{{(0+1)}^2(0−2)}=3\) To find the \(x\)-coordinates of the \(x\)-intercepts, we determine when the numerator of the function is zero. In this case, the graph is approaching the horizontal line \(y=0\). Compare the degrees of the numerator and the denominator to determine the horizontal or slant asymptotes. Answer Horizontal asymptote at \(y=\frac{1}{2}\). Given a rational function, identify any vertical asymptotes of its graph Factor the numerator and denominator. Occasionally, a graph will contain a hole: a single point where the graph is not defined, indicated by an open circle. Figure \(\PageIndex{10}\). As \(x\rightarrow \pm \infty\), \(f(x)\rightarrow 3\), resulting in a horizontal asymptote at \(y=3\). Example: \(f(x)=\dfrac{3x^2+2}{x^2+4x−5}\) In this case, the end behavior is \(f(x)≈\dfrac{3x^2}{x^2}=3\). Figure \(\PageIndex{2}\). Note that we have no method to find the exact location of the local minimum, but by testing a few \(x\)-values, we might notice that \(f(1)=1.5\), so there is a local minimum somewhere between \(x=0\) and \(x=2\). The \(y\)-intercept is \((0,–0.6)\), the \(x\)-intercepts are \((2,0)\) and \((–3,0)\). See Figure \(\PageIndex{17}\). Figure \(\PageIndex{7}\). After passing through each of the \(x\)-intercepts, the graph will eventually tend to a \(y\)-value of zero as \(x \rightarrow -\infty\) and as \(x \rightarrow \infty\), as indicated by the horizontal asymptote. \(h(x)=\frac{x^2−4x+1}{x+2}\): The degree of \(p=2\) and degree of \(q=1\). See Figure \(\PageIndex{18}\). Figure \(\PageIndex{23}\) Now that we have analyzed some important equations for rational functions and how they relate to a graph of the function, we can use information given by a graph to write a possible function. Solution: Shifting the graph left 2 and up 3 would result in the function \[f(x)=\dfrac{1}{x+2}+3 onumber\] or equivalently, by giving the terms a common denominator, \[f(x)=\dfrac{1}{x+2} + \dfrac{3(x+2)}{x+2} = \dfrac{3x+7}{x+2} onumber\] The graph of the shifted function is displayed in Figure \(\PageIndex{7}\). As the values of \(x\) approach infinity, the function values approach \(0\). \(f(x)=\dfrac{1}{{(x−3)}^2}−4=\dfrac{1−4{(x−3)}^2}{{(x−3)}^2}=\dfrac{1−4(x^2−6x+9)}{(x−3)(x−3)}=\dfrac{−4x^2+24x−35}{x^2−6x+9}\) Because the numerator is the same degree as the denominator we know that as \(x\rightarrow \pm \infty\), \(f(x)\rightarrow −4\); so \(y=–4\) is the horizontal asymptote. We call such a hole a removable discontinuity. We have already seen a removable discontinuity, in Figure 4.8.9, at \(x=-3\). For example, the function \(f(x)=\frac{x^2−1}{x^2−2x−3}\) may be re-written by factoring the numerator and the denominator. The horizontal asymptote will be at the ratio of these values: \[\text{As } t\rightarrow \infty,\space C(t)\rightarrow \frac{1}{10} onumber\] This function will have a horizontal asymptote at \(y=\frac{1}{10}\). This occurs when \(x+1=0\) and when \(x - 2=0\), giving us vertical asymptotes with equations \(x=-1\) and \(x=2\). Several things are apparent if we examine the graph of \(f(x)=\frac{1}{x}\). Note that this graph crosses the horizontal asymptote. Setting each factor equal to zero, we find \(x\)-intercepts at \(x=–2\) and \(x=3\). Identify the horizontal and vertical asymptotes of the graph, if any. Use arrow notation to describe the end behavior and local behavior of the function graphed in Figure \(\PageIndex{6}\). A reciprocal function cannot have values in its domain that cause the denominator to equal zero. Solution: Factor the numerator and the denominator. By looking at the graph of a rational function, we can investigate its local behavior and easily see whether there are asymptotes. This line is a slant asymptote. Since the graph has no \(x\)-intercepts between the vertical asymptotes, and the \(y\)-intercept is positive, we know the function must remain positive between the asymptotes, letting us fill in the middle portion of the graph as shown in Figure \(\PageIndex{21}\). Based on this overall behavior and the graph, we can see that the function approaches 0 but never actually reaches 0; it seems to level off as the inputs become large. Example: \(f(x)=\dfrac{3x^2−2x+1}{x−1}\) In this case, the end behavior is \(f(x)≈\frac{3x^2}{x}=3x\). See Figure \(\PageIndex{11}\). We can use this information to write a function of the form \(f(x)=a\dfrac{(x+2)(x−3)}{(x+1){(x−2)}^2}\) To find the stretch factor, we can use another clear point on the graph, such as the \(y\)-intercept \((0,–2)\). DOMAIN OF A RATIONAL FUNCTION The domain of a rational function includes all real numbers except those that cause the denominator to equal zero. At the \(x\)-intercept \(x=3\) corresponding to the \((x−3)\) factor of the numerator, the graph passes through the axis as we would expect from a linear factor. Notice that horizontal and vertical asymptotes are shifted left 2 and up 3 along with the function. Degree of numerator is greater than degree of denominator by one: no horizontal asymptote; slant asymptote. Table \(\PageIndex{2}\). \(\PageIndex{8}\) Given the function \(f(x)=\frac{{(x+2)}^2(x−2)}{2{(x−1)}^2(x−3)}\), use the characteristics of polynomials and rational functions to describe its behavior and sketch a possible graph of the function. In Example \(\PageIndex{2}\), we shifted a toolkit function in a way that resulted in the function \(f(x)=\frac{3x+7}{x+2}\). Find the ratio of freshmen to sophomores at 1 p.m. Answer \(\frac{12}{11}\) A vertical asymptote indicates a value at which a rational function is undefined, so that value is not in the domain of the function. Figure \(\PageIndex{1}\). Next, we set the denominator equal to zero, and find that the vertical asymptote is \(x=3\), because as \(x\rightarrow 3\), \(f(x)\rightarrow \infty\). \(\PageIndex{7}\) Given the reciprocal squared function that is shifted right 3 units and down 4 units, write this as a rational function. Example \(\PageIndex{6}\): Identifying Vertical Asymptotes and Removable Discontinuities for a Graph Find the vertical asymptotes and removable discontinuities of the graph of \(k(x)=\frac{x−2}{x^2−4}\). Solution: Notice that the graph is showing a vertical asymptote at \(x=2\), which tells us that the function is undefined at \(x=2\). As the values of \(x\) approach negative infinity, the function values approach \(0\). Figure \(\PageIndex{17}\). Analysis To find the horizontal asymptote, divide the leading coefficient in the numerator by the leading coefficient in the denominator: \(\dfrac{1}{10}=0.1 onumber\] In this problem, the horizontal asymptote is \(y= 0.1.\) This means the concentration \(C,\) the ratio of pounds of sugar to gallons of water, will approach 0.1 in the long term. Finally, on the right branch of the graph, the curves approaches the \(x\)-axis \((y=0 \) as \(x\rightarrow \infty\). See Figure \(\PageIndex{14}\). Since the multiplicities of \((x-2)\) in both the numerator and denominator are equal, this is the location of a removable discontinuity. Also, although the graph of a rational function may have many vertical asymptotes, the graph will have at most one horizontal asymptote. \(f(0)=\dfrac{(0−2)(0+3)}{(0−1)(0+2)(0−5)}\) The \(x\)-intercepts will occur when the function is equal to zero: \(0=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)}.\) This is zero when the numerator is zero. \[\begin{align*} C(12) & =\dfrac{5+12}{100+10(12)} \\ & = \dfrac{17}{220} \end{align*}\] This means the concentration is 17 pounds of sugar to 220 gallons of water. The one at \(x=-1\) exhibits the basic behavior similar to \(\dfrac{1}{x}\), with the graph heading toward positive infinity on one side and heading toward negative infinity on the other. Since \(p>q\) by 1, there is a slant asymptote \(y=x-6\). For example, the graph of \(f(x)=\dfrac{{(x+1)}^2(x−3)}{{(x+3)}^2(x−2)}\) is shown in Figure \(\PageIndex{20}\). As \(x\rightarrow \pm \infty\), \(f(x)\rightarrow 3\). See Figure \(\PageIndex{13}\). Solution: We start by noting that the function is already factored, saving us a step. Note any restrictions in the domain of the function. See Figure \(\PageIndex{3}\). As \(x\rightarrow 3\), \(f(x)\rightarrow \infty\), and as \(x\rightarrow \pm \infty\), \(f(x)\rightarrow −4\). At the \(x\)-intercept \(x=−1\) corresponding to the \({(x+1)}^2\) factor of the numerator, the graph "bounces", consistent with the quadratic nature of the factor. Example \(\PageIndex{11}\): Graphing a Rational Function Sketch a possible graph of \(f(x)=\dfrac{(x+2)(x−3)}{(x+1)^2(x−2)}\). Answer Figure \(\PageIndex{8}\). Solution: Both the numerator and denominator are linear (degree 1). The end behavior of the graph will mimic the behavior of the reduced end behavior fraction. Figure \(\PageIndex{16}\). When the degree of the factor in the denominator is odd, the distinguishing characteristic will be like the function \(\dfrac{1}{x}\). On one side of the vertical asymptote the graph heads towards positive infinity, and on the other side the graph heads towards negative infinity. The zero of this factor, \(x=−1\), is the location of the removable discontinuity. A tap will open pouring 10 gallons per minute of water into the tank at the same time sugar is poured into the tank at a rate of 1 pound per minute. Note any values that cause the denominator to be zero in this simplified version. \(\PageIndex{4}\) Find the domain of \(f(x)=\dfrac{4x}{5(x−1)(x−5)}\). Since the water increases at 10 gallons per minute, and the sugar increases at 1 pound per minute, these are constant rates of change. Degree of numerator is equal to degree of denominator: horizontal asymptote at ratio of leading coefficients, \(y=\dfrac{a_n}{b_n}\). A rational function written in factored form will have an \(x\)-intercept where each factor of the numerator is equal to zero. Finally, we evaluate the function at 0 and find the \(y\)-intercept to be at \((0,−\frac{35}{9})\). The graph heads toward positive infinity as the inputs approach the asymptote on the right, so the graph will head toward positive infinity on the left as well.

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